Fatigue of Material
Part I

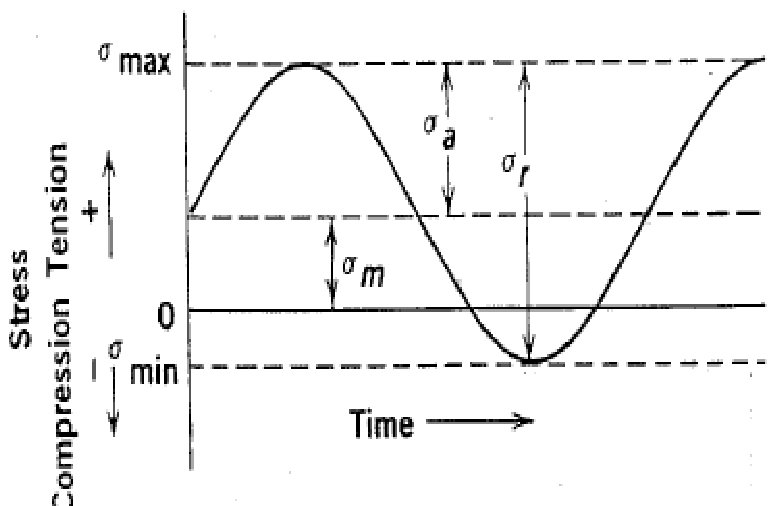
Lecture 6

Fatigue of Materials

- Fatigue is the process by which most materials fail under cyclic loading (~ 80-90%)
- It involves the initiation and evolution of cracks at loads that are too low to cause failure under monotonic loading
- It may occur in implants during normal service

  - heart valves
  - dental implants

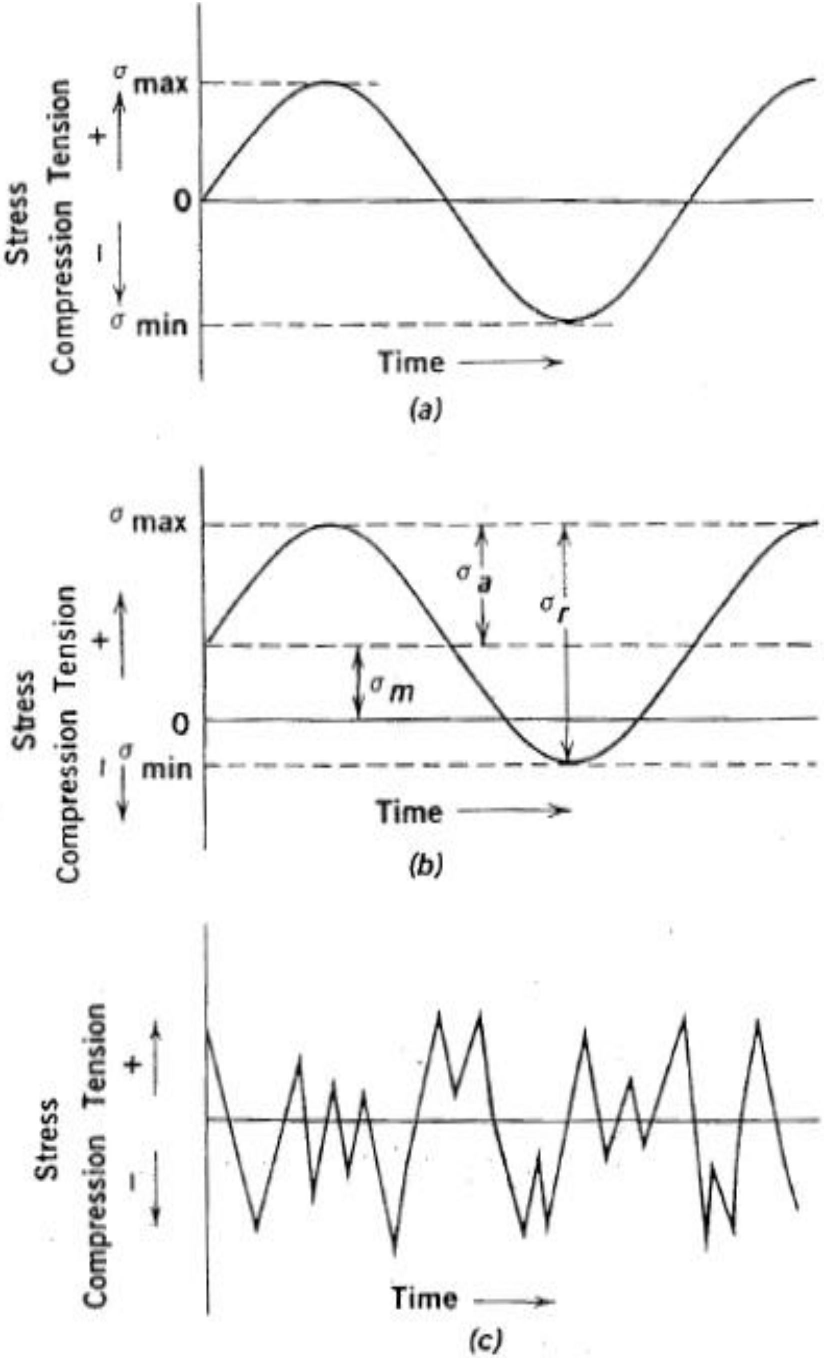
Schematic of Cyclic Loading Profile



$$\sigma_r = \sigma_{max} - \sigma_{min} \qquad \sigma_a = \frac{\sigma_r}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad R = \frac{\sigma_{min}}{\sigma_{max}}$$

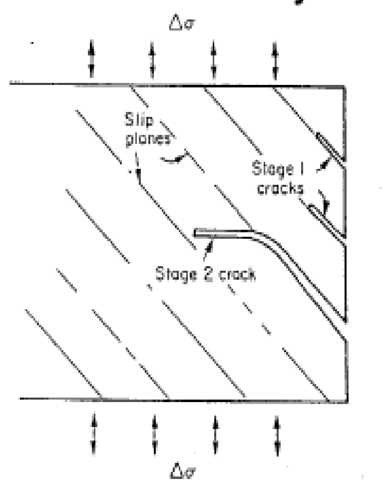
Fatigue of Material



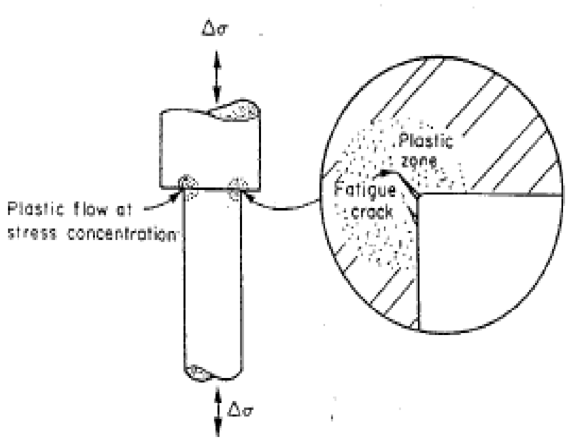
Fatigue Mechanisms

- <u>Two steps</u>: (a) Crack Initiation
  (b) Crack Propagation

- **Crack Initiation in Low-Cycle Fatigue:**

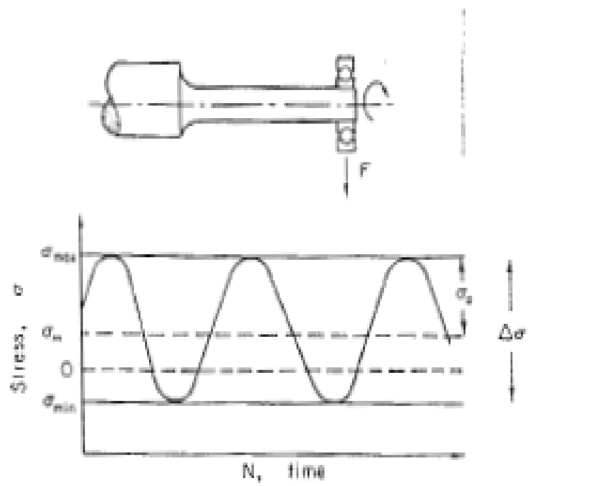


- **Cracks Initiation in High-Cycle Fatigue:**

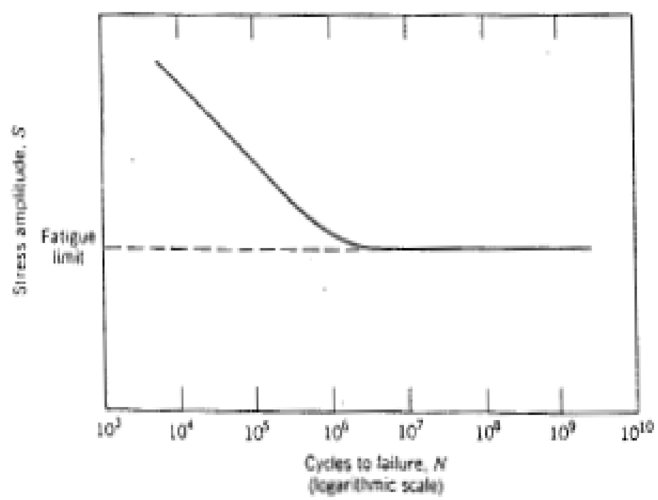


Characterization of Fatigue

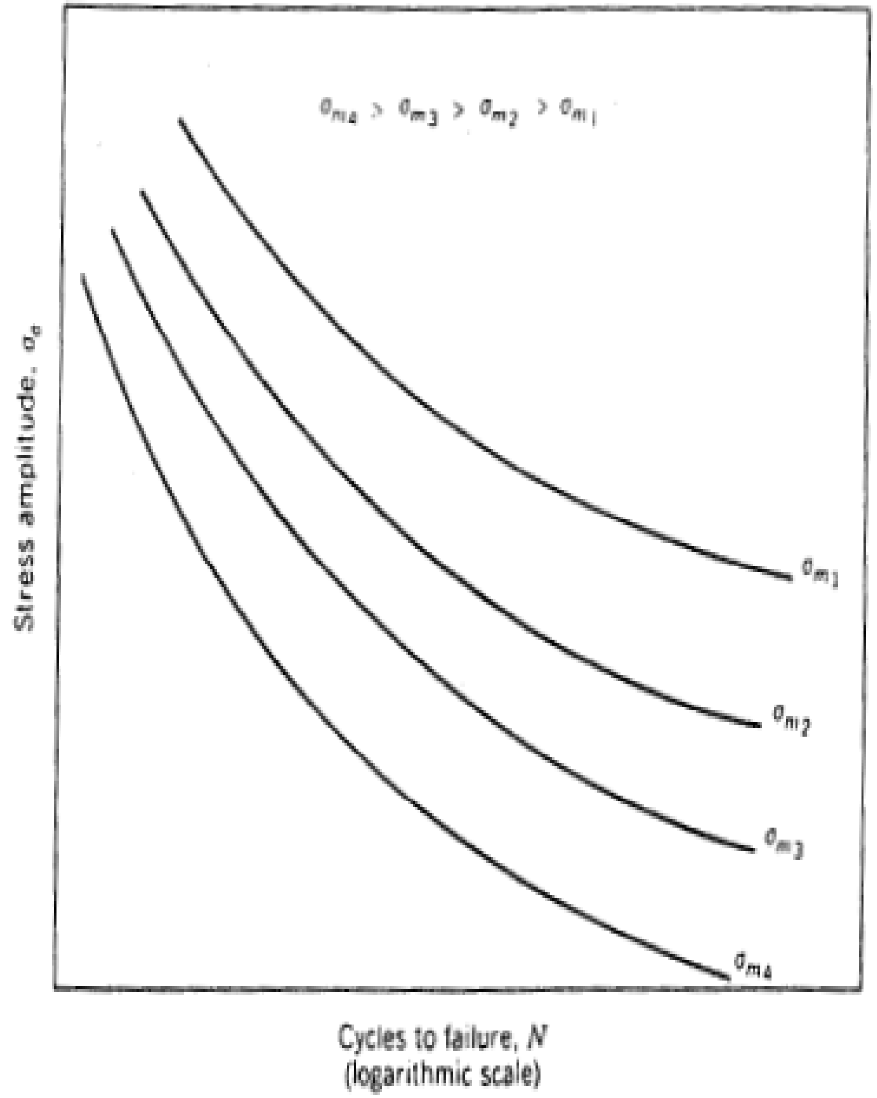
- Types of testing geometries (duplicate in-service loading conditions):



- The Stress Amplitude-Numer of Cycles (S-N) Curve:



Characterization of Fatigue



Demonstration of influence of mean stress σ_m on S–N fatigue behavior.

The Driving Force for Fatigue

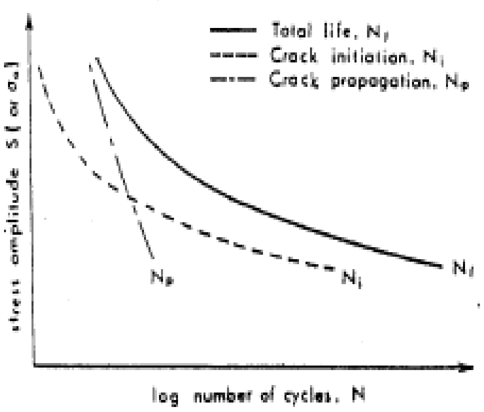
- Fatigue is generally controlled by at least two parameters

  - stress range or strain range
  - mean stress or mean strain

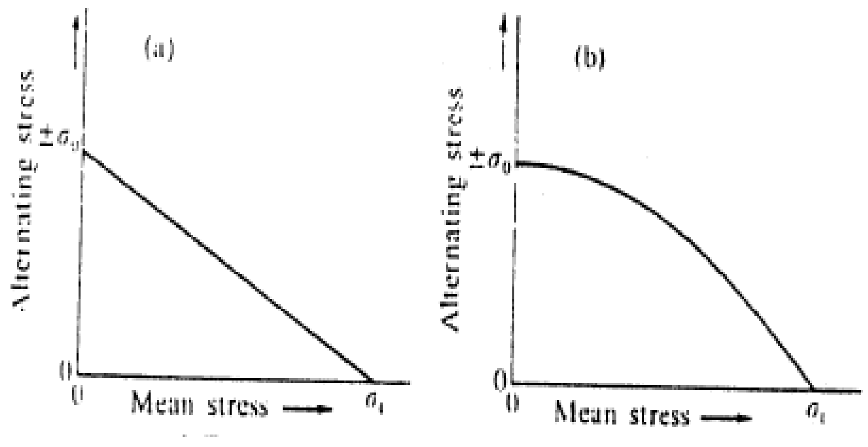
- The overall effects of stress or strain range well characterized by stress-life curves

- The effects of mean stress are well characterized by the Goodman line or Gerber parabola

Stress-Life Curves



Mean Stress Effects



a) Goodman line b) Gerber parabola.

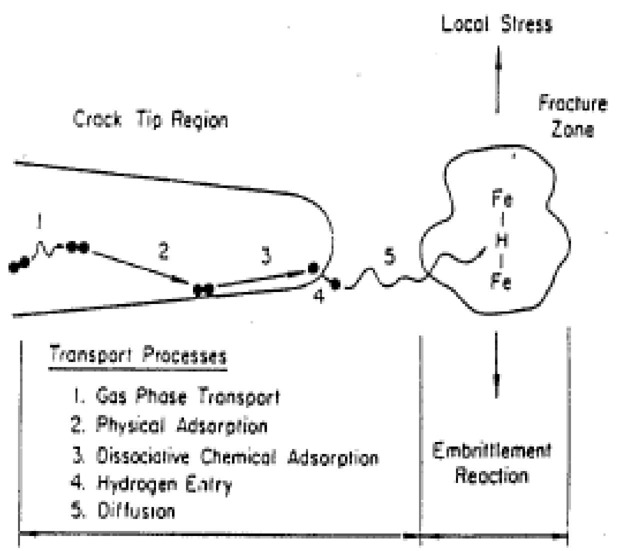
The Factors that Affect Fatigue Life

- Fatigue life is also affected by other parameters

- These include the effects of

  - environment (vacuum, relative humidity, corrosion)
  - surface finish
  - creep and time-dependent phenomena
  - wear under contact fatigue scenarios e.g. in dental implants

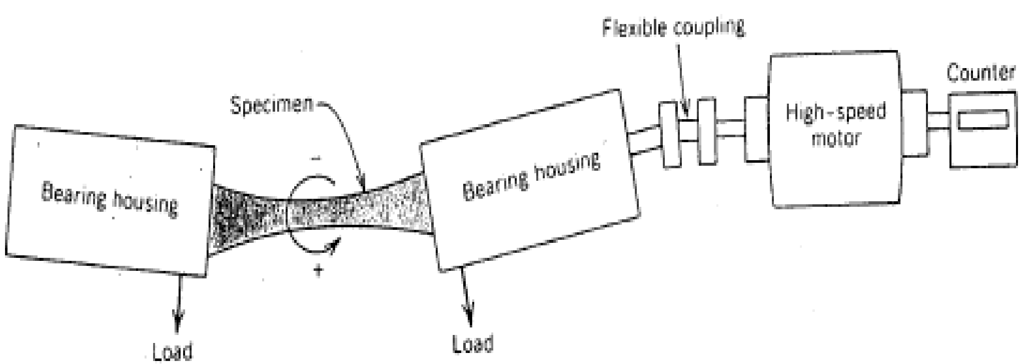
<u>Transport and Chemisorption of Water Vapor</u>



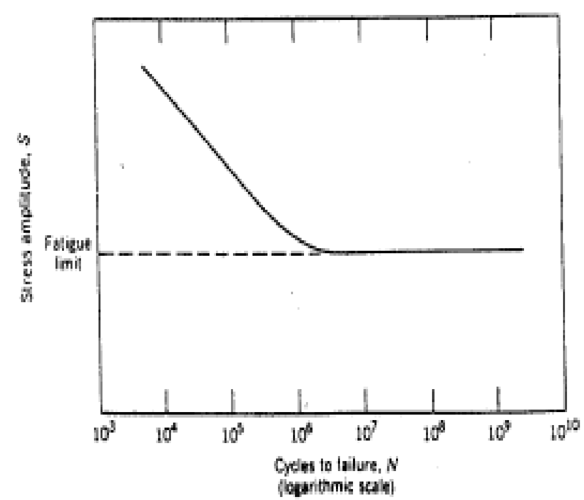
Measurement of Stress-Life Behavior

- The dependence of fatigue life, N_f, on the stress range, ΔS, is often presented on S-N curves
- These were first proposed by Wohler in ~ 1860 and are still used for the characterization of fatigue life
- Some materials exhibit infinite fatigue lives at stresses below the fatigue limit - but others do not

Measurement of Stress-Life Behavior



Stress-Life Behavior



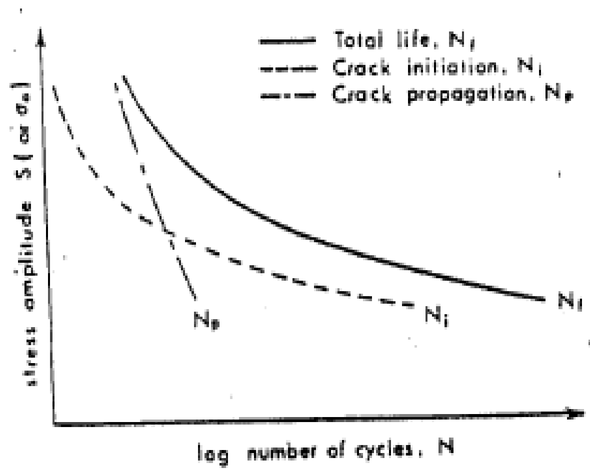
The Stages of Fatigue Damage in Materials

- In most cases - can divide the total fatigue life into two components

  - N_i = initiation life
  - N_p = propagation life
  - N_f = $N_i + N_p$

- Fracture mechanics approach assumes that there are pre-existing cracks - hence $N_I = 0$

Schematic of the Components of Fatigue Life



The Fracture Mechanics Approach to Fatigue

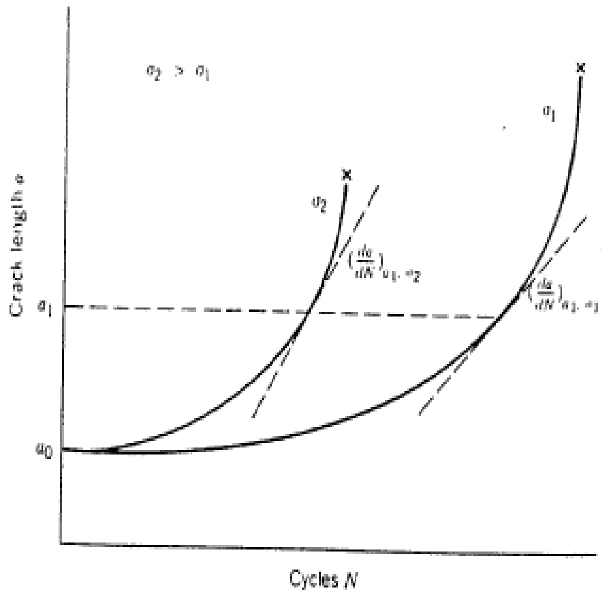
- The fracture mechanics approach assumes that there are pre-existing cracks
- The driving force for fatigue crack growth is the stress intensity factor range

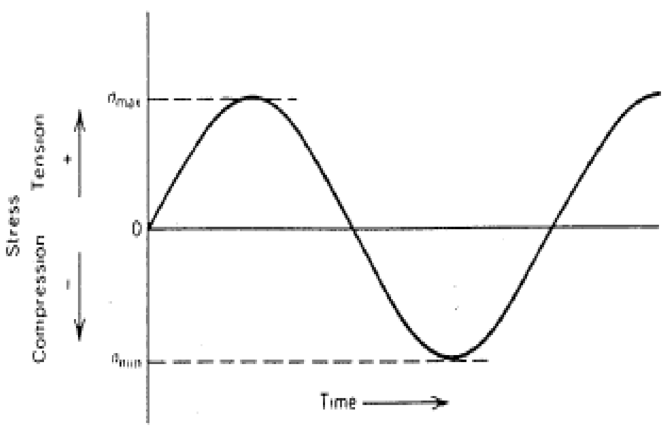
$$\Delta K = K_{max} - K_{min}$$

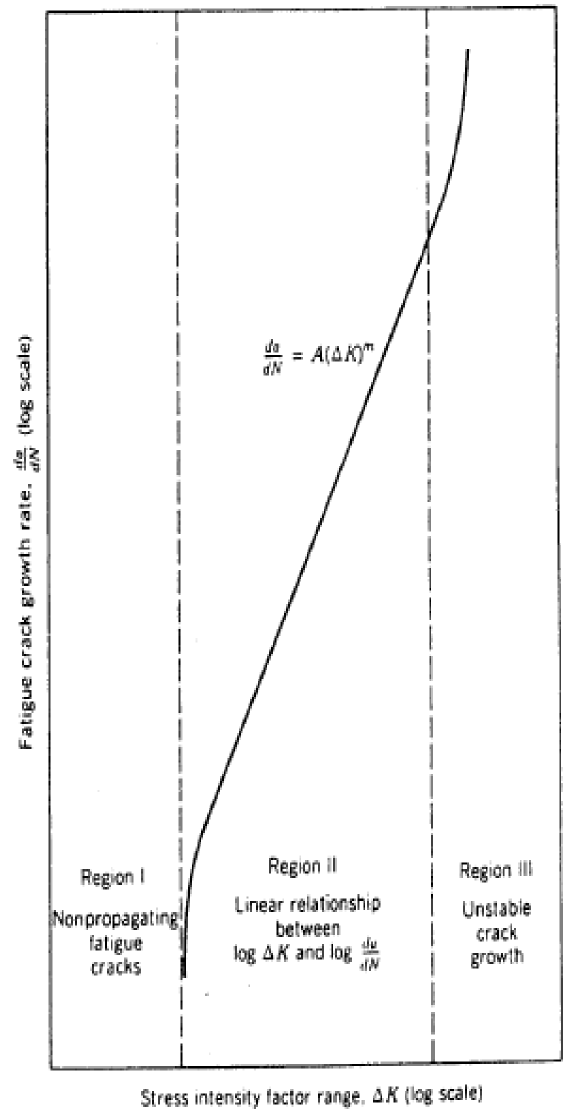
- The crack growth rate is also affected by at least one other parameter

  - $R = K_{min}/K_{max}$
  - $K_{mean} = \dfrac{K_{max} + K_{min}}{2}$
  - K_{max}

The Fracture Mechanics Approach to Fatigue







$$\Delta \mathbf{K} = \mathbf{K}_{max} - \mathbf{K}_{min}$$

$$\Delta \mathbf{K} = \mathbf{Y}\ \Delta\sigma\sqrt{(\pi a)} = \mathbf{Y}(\sigma_{max} - \sigma_{min})\sqrt{(\pi a)}$$

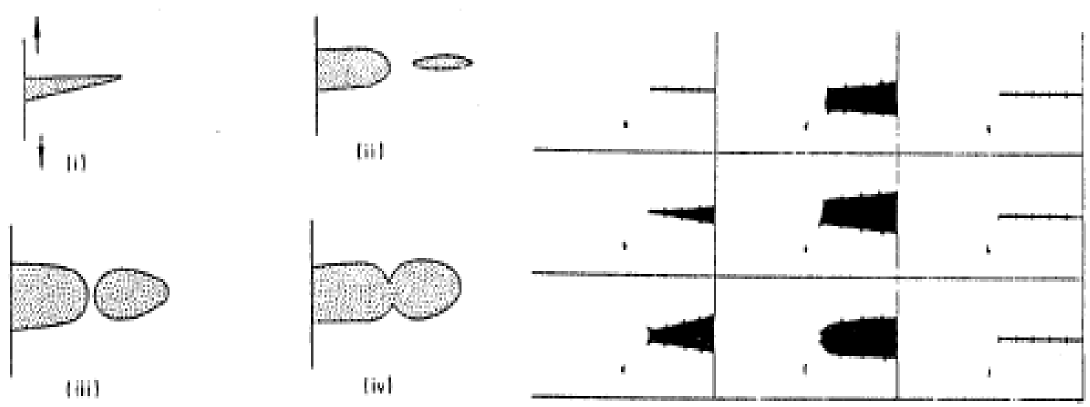
Relationships Between Crack Growth Rates and Crack Driving Forces

- The crack growth rate was first related to crack driving forces by Paris (~ 1956)

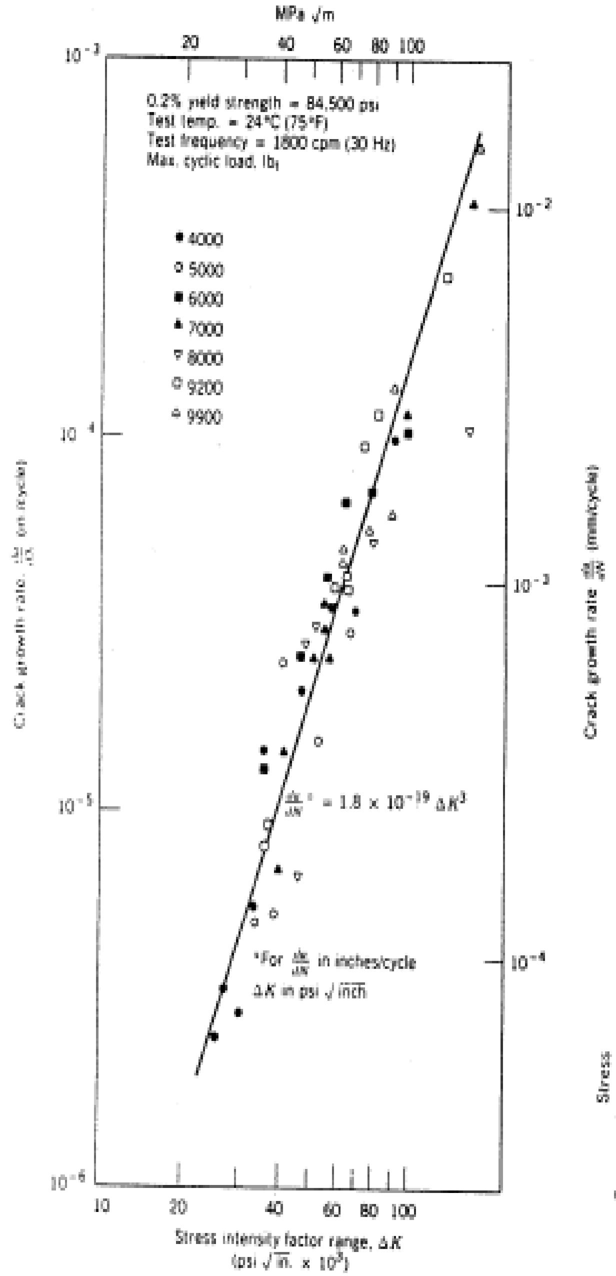
  - $\frac{da}{dN} = \alpha(\Delta K, K_{max})$ etc

- This was later developed into the so-called Paris law

$$\frac{da}{dN} = A(\Delta K)^m$$



Relationships Between Crack Growth Rates and Crack Driving Forces



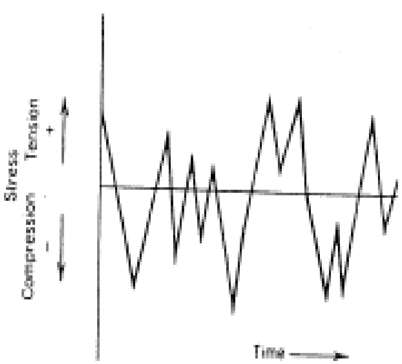
Logarithm crack growth rate versus logarithm stress intensity factor range for a Ni–Mo–V steel. (Reprinted by permission of the Society for Experimental Mechanics, Inc.)

$$N_f = \int_0^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{A(\Delta K)^m}$$

$$N_f = \int_{a_0}^{a_c} \frac{da}{A(Y\,\Delta\sigma\sqrt{\pi a})^m}$$

$$= \frac{1}{A\pi^{m/2}(\Delta\sigma)^m}\int_{a_0}^{a_c}\frac{da}{Y^m a^{m/2}}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma Y}\right)^2$$

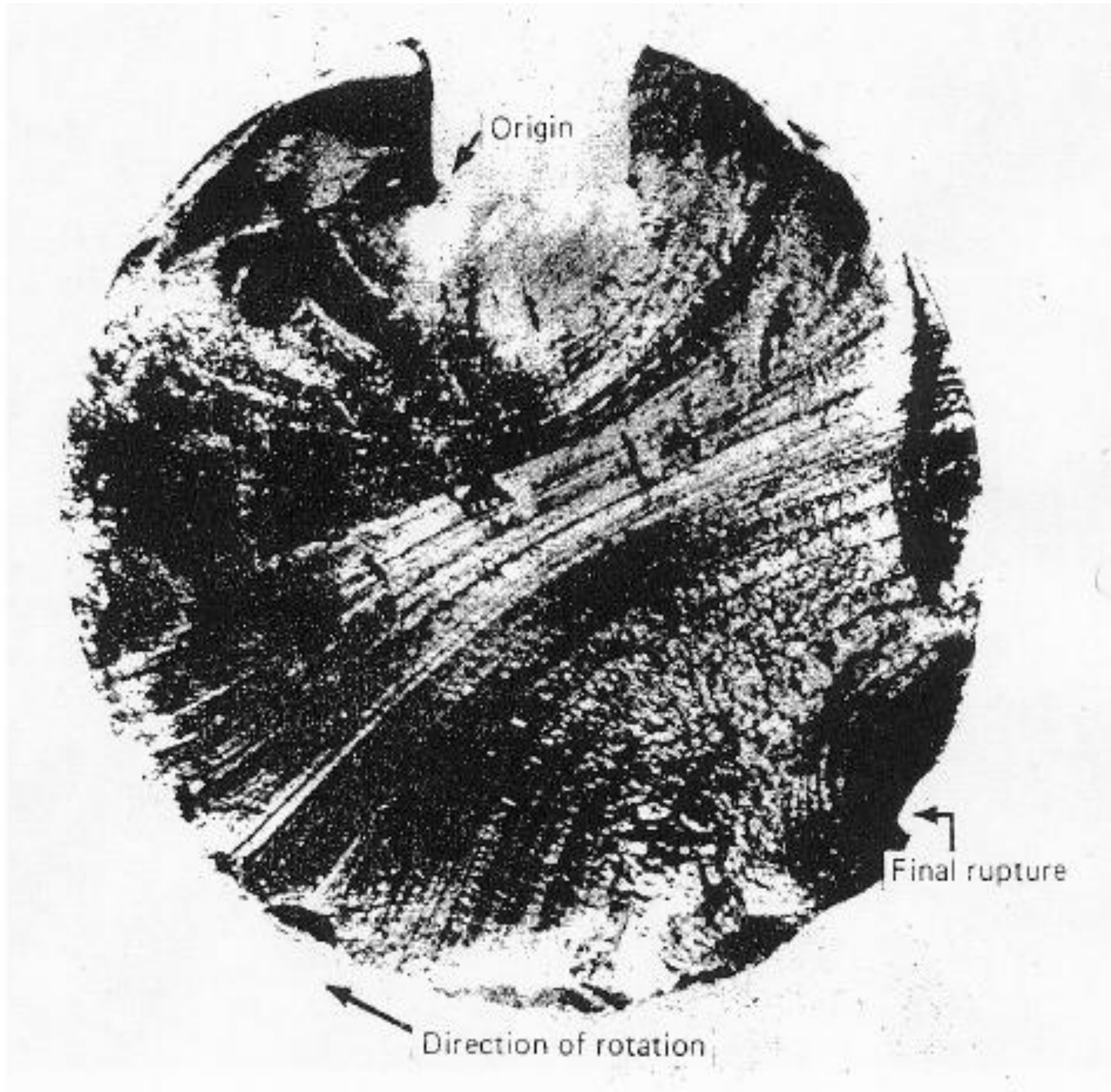


Life Prediction within a Fracture Mechanics Framework

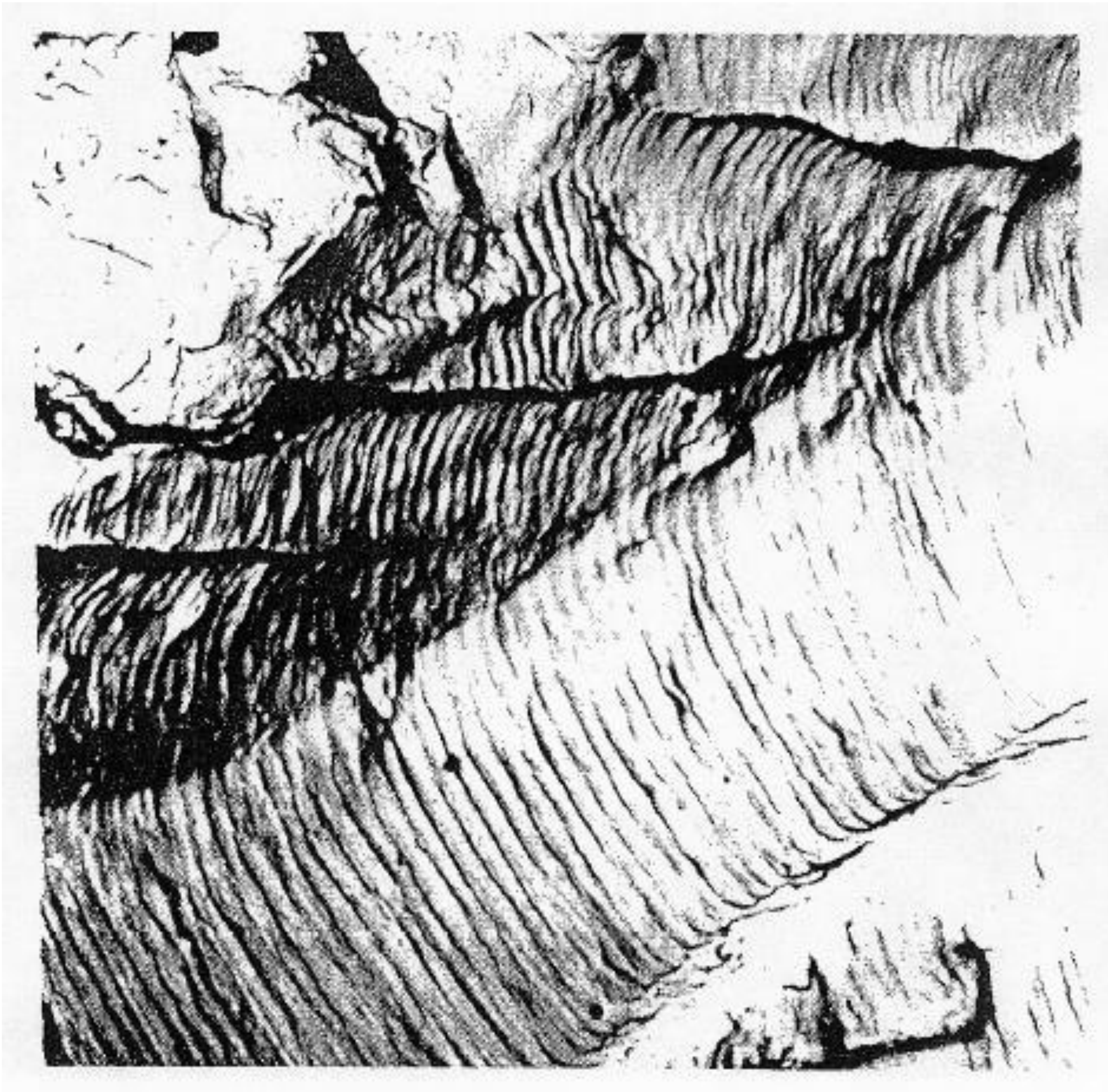
- The fracture mechanics approach can account for the effects of pre-existing cracks
- Life prediction can be achieved by the separation of variables and integration between appropriate limits

General Framework	Example
$\frac{da}{dN} = f(\Delta K, K_{max} \cdots)$	$\frac{da}{dN} = A(\Delta K)^m$
$da = f(\Delta K, K_{max} \cdots) dN$	$da = A(\Delta K)^m \, dN$
$\int_{a_o}^{a_c} da = \int_{o}^{N_f} f(\Delta K, K_{max} \cdots) dN$	$\int_{a_o}^{a_c} da = \int_{o}^{N_f} A(\Delta K)^m dN$

Propagation of Failure



Propagation of Failure



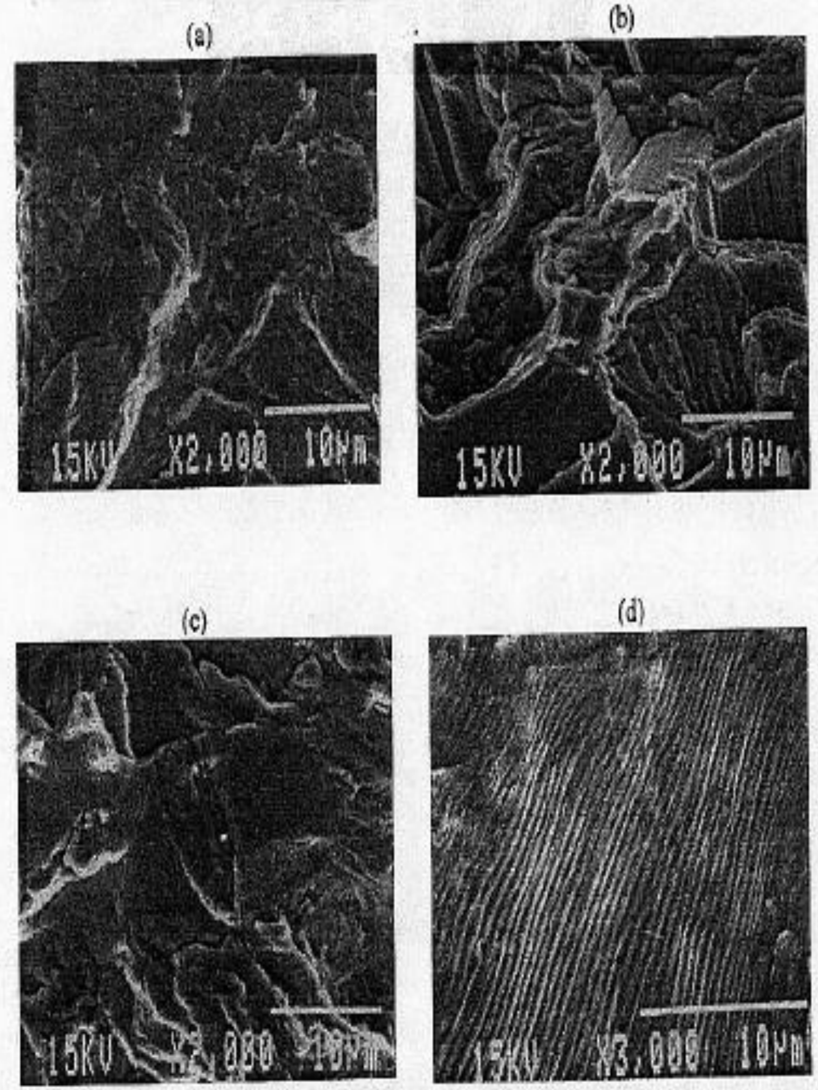
Mechanisms of Fatigue Crack Grwoth

- The diagnosis of fatigue requires an understanding of how to interpret fracture modes
- The fracture modes change with ΔK and K_{max}
- In general, the mechanisms vary from low ΔK, mid ΔK and high ΔK

  - low ΔK (usually crystallographic)
  - mid ΔK (striations)
  - high ΔK striation + dimples

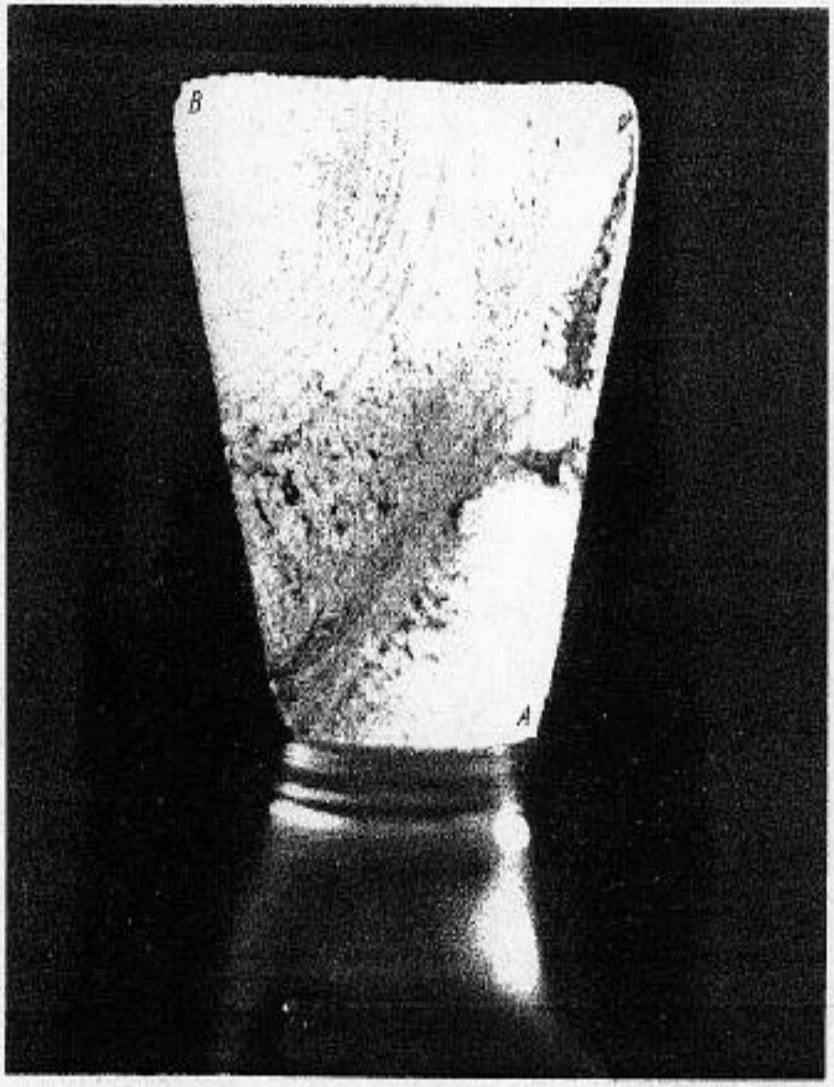
Mechanisms of Fatigue Crack Growth

In Regimes ABC



Dependence of fatigue fracture modes on crack length, a, and stress intensity factor range, ΔK (at $R = 0.1$): (a) $a \approx 50$ µm, $\Delta K \approx 4.5$ MPa m$^{1/2}$; (b) $a \approx 250$ µm, $\Delta K \approx 15.8$ MPa m$^{1/2}$; (c) $a \approx 750$ µm, $\Delta K \approx 25.1$ MPa m$^{1/2}$; and (d) $a \approx 1.6$ mm, $\Delta K \approx 34.8$ MPa m$^{1/2}$. The crack has grown from left to right in all the figures.

Crack Growth From Posteromedial Corner



Fatigue Striations on Surface of Lateral Crack